CNF Encodings of Binary Constraint Trees (Extended Abstract)

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Abstract

Ordered Multi-valued Decision Diagrams (MDDs) have been shown to be useful to represent finite domain functions/relations. For example, various constraints can be modelled with MDD constraints. Recently, a new representation called Binary Constraint Tree (BCT), which is a (special) tree structure binary Constraint Satisfaction Problem, has been proposed to encode MDDs and shown to outperform existing MDD constraint propagators in Constraint Programming solvers. BCT is a compact representation, and it can be exponentially smaller than MDD for representing some constraints. Here, we also show that BCT is compact for representing non-deterministic finite state automaton (NFA) constraints. In this paper, we investigate how to encode BCT into CNF form, making it suitable for SAT solvers. We present and investigate five BCT CNF encodings. We compare the propagation strength of the BCT CNF encodings and experimentally evaluate the encodings on a range of existing benchmarks. We also compare with seven existing CNF encodings of MDD constraints. Experimental results show that the CNF encodings of BCT constraints can outperform those of MDD constraints on various benchmarks.

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1 Introduction

Ordered Multi-valued Decision Diagram (MDD) [14] is a compact representation which can be used to encode finite domain functions/relations. Many constraints can be encoded into compact MDD constraints, such as the regular constraints [12], table constraints [4], among and sequence constraints [9]. MDD constraints are also useful to model problems requiring specific constraints which are not readily modelled with existing known constraints [5]. In Constraint Programming (CP) solvers, MDD constraints can be directly handled with Generalized Arc Consistency (GAC) propagators. Alternatively, MDD constraints can also be solved by SAT solvers by encoding MDD constraints into CNF form [1]. In this way, SAT solvers can directly handle the constraints which can be modelled with MDDs constraints.

Binary constraint is also a general representation for constraints. Any non-binary constraint can be transformed into binary constraints through binary encodings such as dual encoding [6], hidden variable encoding [13], double encoding [15] and bipartite encoding [18]. Recently, binary encodings with specialized Arc Consistency (AC) propagators [17, 18] has been shown to outperform the GAC propagators of non-binary table constraints [20]. Similar to MDDs, the binary constraints can also be encoded into CNF with different CNF encodings, such as the log encoding [16], direct encoding [16] and support encoding [8].

Recently, a new representation called Binary Constraint Tree (BCT) [19], which is a set of binary constraints with tree structures (a special binary CSP), has been proposed to encode MDDs. BCT is a compact representation, and it can be exponentially smaller than MDD. In

	Log	Direct	Minimal Support	Partial Support	Support
Weak consistency	✗	✗	✓	✓	✓
Domain consistency	✗	✗	✗	✓	✓
Unit refutation completeness	✗	✗	✓	✓	✓
Propagation completeness	✗	✗	✗	✗	✓

Table 1 Strength of Unit Propagation on various encodings of BCT constraints. The label ✓(✗) denotes CNF encodings implement (does not implement) a certain unit propagation strength level.

this paper, we also show that non-deterministic finite state automaton (NFA) constraints [3] can be transformed into BCT constraints without exponential blow up but not MDD constraints. Furthermore, a GAC propagator of BCT constraints [19] has been shown to outperform the state-of-the-art MDD GAC propagators. The results in [19] show that BCT has great potential for encoding and reducing MDDs. In this work, we investigate how to encode BCT constraints into CNF instances and apply them in SAT solvers.

2 Main contributions

We investigate five CNF encodings of BCT constraints using the well-known log, direct, support encodings and our new encodings: partial support encoding and minimal support encoding. Then we analyze the strength of unit propagation on these five CNF encodings. We consider 4 propagation levels: weak consistency, domain consistency, unit refutation completeness [7, 1, 10, 11] and propagation completeness [2, 1, 10, 11], where propagation completeness is the strongest level, unit refutation completeness is incomparable with domain consistency, and weak consistency is the weakest level.

Table 1 summarizes the strength of unit propagation on all five CNF encoding of BCT constraints. The support encoding of BCT constraints, which implements propagation completeness, can have a greater propagation strength than the other CNF encodings. The partial support encoding and minimal support encoding are more compact than the support encoding but their propagation strength is weaker than the support encoding. The partial support encoding implementing domain consistency is stronger than the log, direct and minimal support encodings. In addition, the log and direct encodings of BCT constraints are weaker than the minimal support encoding, since the log and direct encodings of BCT constraints do not implement weak consistency.

We evaluate our five CNF encodings of BCT constraints with seven existing CNF encodings [1] of MDD constraints and also a BCT GAC propagator [19] on a range of existing benchmarks. Our experimental results show that the CNF encodings of BCT constraints can outperform those of MDD constraints. Our results show that solving of BCT constraints as well as NFA/MDD constraints is promising on SAT solvers. While there is some initialization and encoding time for all methods, this is overall less significant than the solving time. The initialization time becomes significant when the encoding becomes large. For example, on some instances, the encoding cost becomes significant in the MDD CNF encodings with some being memory-out. BCT CNF encodings generally outperform MDD CNF encodings. As with the MDD CNF encoding experiments in [1] where they found performance was mixed between CNF encodings and their propagator comparison, we also find that for some problems the BCT CNF encoding is the best while for other problems the BCT propagator in Abscon is the best. BCT CNF encoding is overall competitive or best for many instances and increases the flexibility and choices in solving of BCT (and NFA/MDD) constraints.

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