# Solving the Constrained Single-Row Facility Layout Problem with Decision Diagrams (Extended Abstract)

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## Abstract

This paper presents two exact optimization models for the Constrained Single-Row Facility Layout Problem. It is a linear arrangement problem considering departments in a facility with given lengths and traffic intensities. The first approach is an extension of the state-of-the-art mixed-integer programming model for the unconstrained problem with the additional constraints. The second one is a decision diagram-based branch-and-bound that takes advantage of the recursive nature of the problem through a dynamic programming model. The computational experiments show that both approaches significantly outperform the only mixed-integer programming model in the literature.

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## 1 Introduction

Given a set of departments $N = \{1, \ldots, n\}$ in a facility, their lengths $l_i$ and pairwise traffic intensities $c_{ij}$, the *Constrained Single-Row Facility Layout Problem* (cSRFLP) [9] aims to find a linear arrangement $\pi : N \to N$ that minimizes the total distance travelled in the facility. If $d_{ij}^{\pi}$ denotes the distance between departments $i$ and $j$, the objective function can be formulated as $cSRFLP(\pi) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} c_{ij} d_{ij}^{\pi}$. The arrangement is subject to three types of constraints described below:

- Positioning constraints $position : N \to N \cup \{0\}$ map departments to a forced position, or 0 when unconstrained. The function $department : N \to N \cup \{0\}$ is the inverse mapping.
- Ordering constraints $predecessors : N \to 2^N$ specify a set of departments that must appear before each department in the arrangement.
- Relation constraints $previous : N \to N \cup \{0\}$ impose a department to be placed right after another one.

In this paper, we present two approaches for the cSRFLP and show that they significantly outperform the *mixed-integer programming* (MIP) model presented in [10].

## 2 Mixed-Integer Programming Model

The cSRFLP is an extension of the well-known *Single-Row Facility Layout Problem* (SRFLP) for which the state-of-the-art method is the semi-definite programming approach presented in [8]. This approach would hardly integrate the constraints described above in its formulation. However, we managed to model them in the MIP model from [3] which is also very efficient.

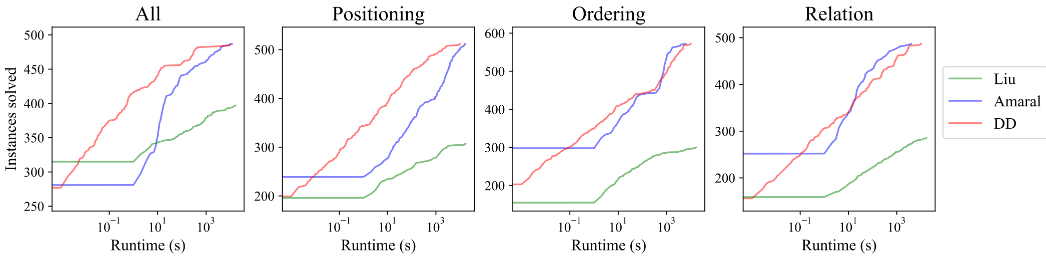


**Figure 1** Number of instances solved by each algorithm for the different types of constraints.

It uses betweenness variables that describe the relative ordering of triplets of departments $i, j, k \in N$: $\zeta^{\pi}_{ijk} = 1$ if $\pi(i) < \pi(k) < \pi(j)$ or $\pi(j) < \pi(k) < \pi(i)$ and $\zeta^{\pi}_{ijk} = 0$ otherwise. To be able to specify absolute ordering constraints, we add two dummy departments $L$ and $R$ located respectively at the far left and right of the arrangement. This allows us to express the constraints using the betweenness variables $\zeta^{\pi}_{Lij}$ and $\zeta^{\pi}_{iRj}$. For instance, a positioning constraint $position(i) = j \neq 0$ can be written as $\sum_{k=1}^{n} \zeta_{Lik} = j - 1$ and $\sum_{k=1}^{n} \zeta_{iRk} = n - j$.

# 3 Decision Diagrams Approach

In [11], a compact *dynamic programming* (DP) model of the SRFLP was introduced. It gives the value of the optimal arrangement $f(N)$ through the recurrence relation $f(S) = \min_{k \in S} f(S \setminus \{k\}) + l_k \sum_{i \in N \setminus S} \sum_{j \in S \setminus \{k\}} c_{ij}$ with $f(\emptyset) = 0$. The departments are added from left to right on the line. When placing department $k$, we compute the total traffic intensity going from departments located on its left to those on its right and we multiply it by the length $l_k$. The constraints of the cSRFLP can be easily integrated by filtering the departments that can be placed next. This DP model can be used to compile *decision diagrams* (DDs) within a branch-and-bound algorithm where branching occurs over the nodes of the DDs [5]. By limiting the width of the DDs, we obtain *relaxed* and *restricted* DDs. They can provide lower and upper bounds for the subproblems. In our approach, however, a heuristic lower bound yielded the best results.

# 4 Computational Experiments

We performed computational experiments to compare the MIP model from Liu et al. [10], the MIP model introduced by Amaral [3] and extended in Section 2 and the DD-based approach presented in Section 3. Both MIP models were implemented with Gurobi version 9.1.2 [6] and the DD approach was implemented in C++. The algorithms are evaluated on classical SRFLP instances taken from [1, 2, 4, 7, 12] with up to 25 departments and to which we added sets of random constraints. Figure 1 shows that Liu fails to solve many instances under the 5h allowed for each while Amaral and DD solve all of them. Moreover, DD is particularly good at handling positioning constraints.

# 5 Conclusion

Two exact approaches for the cSRFLP were presented and shown to have better performance than the only MIP model in the literature. Since we obtained better lower bounds with a heuristic than with relaxed DDs, it would be interesting to continue the search for original relaxation operators or possibly to improve our modeling.

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