Extended Abstract: Sequence Variables for Routing Problems

Augustin Delecluse
TRAIL, ICTEAM, UCLouvain, Belgium

Pierre Schaus
ICTEAM, UCLouvain, Belgium

Pascal Van Hentenryck
Georgia Institute of Technology, USA

Abstract

Constraint Programming (CP) is one of the most flexible approaches for modeling and solving vehicle routing problems (VRP). This paper proposes the *Sequence Variable* domain, that is inspired by the insertion graph introduced in [2] and the subset bound domain for set variables. This domain representation, which targets VRP applications, allows for an efficient insertion-based search on a partial tour and the implementation of simple, yet efficient filtering algorithms for constraints that enforce time-windows on the visits and capacities on the vehicles. Experiment results demonstrate the efficiency and flexibility of this CP domain for solving some hard VRP problems, including the Dial-A-Ride, the Patient Transportation, and the asymmetric TSP with time windows.

2012 ACM Subject Classification Theory of computation → Constraint and logic programming

Keywords and phrases Constraint Programming, Dial-A-Ride, Patient Transportation, TSPTW, Vehicle Routing, Sequence Variables, Insertion Variables

Digital Object Identifier 10.4230/LIPIcs.CP.2022.

Funding *Augustin Delecluse*: This work was supported by Service Public de Wallonie Recherche under grant n°2010235 – ARIAC by DIGITALWALLONIA4.A

This is an extended abstract of the paper accepted at the main conference: Augustin Delecluse, Pierre Schaus, and Pascal Van Hentenryck. Sequence variables for routing problems. In *28th International Conference on Principles and Practice of Constraint Programming (CP 2022)*, 2022.

1 Introduction

Constraint Programming (CP) is one of the most flexible approaches for modeling vehicle routing problems (VRP) [12]. However, the standard model consisting of so-called successor model with one variable for each visit [1] does not allow the insertion of a visit in a partial tour, formed during the search. Furthermore, it fails to represent the optional aspect of some visits without the introduction of a fake vehicle. The goal of the Sequence Variable is to address those two limitations.

1. It can easily model the exclusion of visits not inserted in a tour similarly to a set variable.
2. Inspired by the idea of the *insertion graph* [2], it allows the insertion of a visit in the middle of the partial tour, enabling the implementation of a depth first tree search insertion exploration algorithm similar to the ones used in [2, 7] to reinsert optimally a set of relaxed visits in a large neighborhood search (LNS).

2 Related Work

In [11], the authors introduced a Sequence Variable for scheduling and routing problems. This domain representation directly extends the subset bound domain representation for set variables [6] by partitioning the visits into a required, possible and excluded set plus a partial sequence and a set of insertion points. In this work we simplify this idea by getting rid of the required set. As a consequence, a possible visit must be directly scheduled in the partial sequence but cannot be required without being inserted in the sequence. This modification, despite its simplicity, greatly eases the reasoning made by the constraints and their time complexity, while losing little to no flexibility in practice. The proposed sequence domain can be seen as the making of the *insertion graph* idea introduced in [2] more generic and encapsulated as the internal implementation of the Sequence Variable domain.

3 Sequence Variable

Our Sequence Variable is aimed for VRP, where a vehicle visits a set of location, or nodes. Those nodes can be in one of three state with respect to one Sequence Variable: they can be excluded from it, possibly visited later on, or currently visited. The Sequence Variable allows to change the state of possible nodes, by inserting them into the route, or excluding them. To insert them, each node x stores a set of possible insertions I^x: nodes in the sequence after which they can be inserted. Constraints remove invalid insertions and the sequence is bound whenever no node is still possible: they have all been inserted or excluded.

The domain of this variable is reversible, in order to be used in trail-based solvers such as MiniCP [10], and most updates over the domain are done in constant or linear time. Furthermore, a Sequence Variable can notify constraints on a domain change through three types of hookup events: the sequence becomes bound; a node has been inserted or excluded; the number of insertions I^x for a node has changed. Those events provide the node been changed as a parameter of the notification, to allow incremental updates. During the search, the Sequence Variable can also give for each node x its number of insertions within the route in constant time. This can be used to implement efficiently branching strategies where the nodes with the fewer insertions is branched on first, similarly to [7].

To demonstrate that we can obtain the same flexibility as [11], we have implemented several constraints, including a Transition Times constraint, linking nodes with a time window, and a Cumulative constraint, allowing to model pickup and delivery problems. Those constraints have been used to model three VRP: the Dial-A-Ride Problem (DARP) [4, 7], the Patient Transportation Problem (PTP) [3, 8] and the Traveling Salesman With Time Window (TSPTW). Experiments on those problems show that our approach is competitive with [7] for the DARP, with a slight advantage for the objective on instances with fewer nodes, but not on larger ones. For the PTP, we were able to find new best found solutions on the largest instances compared to [3]. Finally, for the TSPTW, we were able to improve the best found solutions from 32 instances [9]. Some of those solutions could be found within less than a second when combining our Sequence with LNS and insertion-based heuristics.

4 Conclusions and future work

This paper introduced a simplified version of the Sequence domain introduced in [11] as a flexible and effective approach for modeling and solving VRP with CP. Experimental results on three VRP show that our models are competitive with existing sequence based approaches while being effective enough to discover new best solutions to a well-studied problem such as the TSPTW.

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